AFRL-VA-WP-TP-2007-305

NONLINEAR ROBUST/ADAPTIVE CONTROLLER DESIGN FOR AN AIR-BREATHING HYPERSONIC VEHICLE MODEL (PREPRINT)

Lisa Fiorentini, Andrea Serrani, Michael A. Bolender, and David B. Doman

DECEMBER 2006

Approved for public release; distribution unlimited.

STINFO COPY

**AIR VEHICLES DIRECTORATE
AIR FORCE MATERIEL COMMAND
AIR FORCE RESEARCH LABORATORY
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7542**

NOTICE AND SIGNATURE PAGE

AFRL-VA-WP-TP-2007-305 HAS BEEN REVIEWED AND IS APPROVED FOR PUBLICATION IN ACCORDANCE WITH ASSIGNED DISTRIBUTION STATEMENT.

*//Signature//

MICHAEL A. BOLENDER
Aerospace Engineer
Air Force Research Laboratory
Air Vehicles Directorate

//Signature//

DEBORAH S. GRISMER
Chief, Control Design and Analysis Branch
Air Force Research Laboratory
Air Vehicles Directorate

//Signature//

JEFFREY C. TROMP
Senior Technical Advisor
Control Sciences Division
Air Vehicles Directorate

*Disseminated copies will show “//Signature//” stamped or typed above the signature blocks.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
December 2006	Conference Paper Preprint	06/15/2006 – 12/31/2006

4. TITLE AND SUBTITLE
NONLINEAR ROBUST/ADAPTIVE CONTROLLER DESIGN FOR AN AIR-BREATHING HYPERSONIC VEHICLE MODEL (PREPRINT)

5a. CONTRACT NUMBER
F33615-01-2-3154

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
0602201

6. AUTHOR(S)
Lisa Fiorentini and Andrea Serrani (The Ohio State University)
Michael A. Bolender and David B. Doman (AFRL/VACA)

5d. PROJECT NUMBER
A02D

5e. TASK NUMBER

5f. WORK UNIT NUMBER
0A

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

The Ohio State University Collaborative Center for Control Science Department of Electrical and Computer Engineering, 2015 Neil Ave. Columbus, OH 43210	Control Design and Analysis Branch (AFRL/VACA) Control Sciences Division Air Vehicles Directorate Air Force Materiel Command Air Force Research Laboratory Wright-Patterson Air Force Base, OH 45433-7542

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Vehicles Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson Air Force Base, OH 45433-7542

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL-VA-WP

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-VA-WP-TP-2007-305

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
Conference paper submitted to the Proceedings of the 2007 AIAA Guidance, Navigation and Control Conference and Exhibit, published by AIAA.

The U.S. Government is joint author of this work and has the right to use, modify, reproduce, release, perform, display, or disclose the work. PAO Case Number: AFRL/WS 07-0123 (cleared January 22, 2007). Paper contains color.

14. ABSTRACT
This paper describes the design of a nonlinear controller for an air-breathing hypersonic vehicle. To overcome the analytical intractability of this model, a nominal control-oriented model is used to derive the control law. The nominal model has unstable zero dynamics with respect to the output to be controlled, namely flight path angle and velocity, and presents intricate couplings between the engine dynamics and flight dynamics. The flexible effects have not been included in the analysis yet. Adaptive control techniques and robust control techniques are applied to achieve tracking of velocity and altitude trajectories. Simulation results are provided to show that the derived control law allows to achieves excellent tracking performance on the nominal model.

15. SUBJECT TERMS
Flight Control, Hypersonic Vehicles

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Michael A. Bolender
Unclassified	Unclassified	Unclassified	SAR	22	**19b. TELEPHONE NUMBER** *(Include Area Code)* N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

Nonlinear Robust/Adaptive Controller Design for an Air-breathing Hypersonic Vehicle Model

L. Fiorentini*

A. Serrani †

Collaborative Center of Control Science

The Ohio State University, Columbus, OH 43210 USA

Michael A. Bolender‡

David B. Doman §

Air Force Research Laboratory, Wright-Patterson AFB, OH 45433

This paper describes the design of a nonlinear controller for an air-breathing hypersonic vehicle. To overcome the analytical intractability of this model, a nominal control-oriented model is used to derive the control law. The nominal model has unstable zero dynamics with respect to the output to be controlled, namely flight path angle and velocity, and presents intricate couplings between the engine dynamics and flight dynamics. The flexible effects have not been included in the analysis yet. Adaptive control techniques and robust control techniques are applied to achieve tracking of velocity and altitude trajectories. Simulation results are provided to show that the derived control law allows to achieves excellent tracking performance on the nominal model.

I. Introduction

The design of guidance and control systems for air-breathing hypersonic vehicles is a challenging task, due to the high complexity of the equations of motion, stemming form strong couplings between propulsive and aerodynamic effects, significant flexibility associated with the slender geometries required for these aircraft, and large model uncertainty.[1–3] For linearized versions of hypersonic vehicle models, several results are available in the literature,[1,2,4–8] which consider model of various complexity. Tournes et al. employed a nonlinear variable structure control approach in Ref. 9, while several other nonlinear control approaches have been proposed for non-flexible models of more general types of vehicles.[10–13] More recently, in Ref. 14 and 15, linear controllers with explicit constraints on the inputs are derived for linearized models using on-line optimization and anti-windup techniques.

Especially when flexibility effects are taken into account in the model, the equations of motion may become exceedingly complex, due to intricate interactions between the structural, aerodynamic and propulsion systems. As this happens, models derived from first principles can be used effectively only for simulations or validation purposes, whereas the control design must be performed on the basis of control-oriented of reduced complexity which approximates the behavior of a first-principle model to a satisfactory degree.

In Ref. 16, a control oriented model is derived for the longitudinal dynamical model developed by Bolender and Doman[3] using curve fits calculated directly from the forces and moments included in the truth model. Feedback linearization techniques are then applied to the control-oriented model to derive a nonlinear controller. Even though this approach allows to achieve good tracking performance, due to the model complexity, it inevitably leads to very complicated expressions for the hight-order Lie derivatives of the

*Graduate Student, Department of Electrical and Computer Engineering, 2015 Neil Ave.

†Assistant Professor, Department of Electrical and Computer Engineering, 2015 Neil Ave., Member AIAA.

‡Aerospace Engineer, AFRL/VACA, 2210 Eighth St. Suite 21, Senior Member AIAA.

§Senior Aerospace Engineer, AFRL/VACA, 2210 Eighth St. Suite 21, Assoc. Fellow AIAA.

Table 1. States, control inputs, and controlled output for the vehicle model

State		Input	
V	Vehicle velocity	Φ	Fuel-to-air ratio
h	Altitude	A_d	Diffuser area ratio
α	Angle of attack	δ_e	Control surface deflection (elevator)
θ	Pitch angle	δ_c	Control surface deflection (canard)
γ	Flight path angle, $\gamma = \theta - \alpha$		
Q	Pitch rate	Output	
η_i	i-th Generalized elastic coordinate	V	Vehicle velocity
$\dot{\eta}_i$	Time derivative of η_i	h	Altitude

regulated outputs, and ultimately for the controller. Furthermore, it is not possible to perform a robustness analysis when model uncertainty is taken into consideration.

In this study, we take a first step toward a more general nonlinear design, not based on dynamic inversion, for a given hypersonic vehicle model. While a direct application of nonlinear control system design techniques such as backstepping[17] and forwarding [18,19] is not applicable - since the vector field structure is neither in strict feedback form, nor in feed-forward form - some basic ideas from these techniques are used to derive the controller. Following a classic sequential loop closure approach,[20] the model dynamics have been decomposed into subsystems each of which has its own control input. In particular, the thrust is used to control the velocity of the vehicle, the angle of attack is used to control the flight path angle and the altitude, and the moment is then used to control the angle of attack/pitch rate subsystem. In each subsystem the control law is derived using a "mixed" approach based on adaptive techniques[21] and robust techniques such as the small-gain theorem for input-to-state stable systems.[22,23] Since dynamic inversion is not applied, the controller does not depend explicitly on the model coefficients, and can in principle be designed to provide robustness with respect to parameter uncertainties and variations within a given range.

The vehicle model considered in this work is the assumed-modes version[24] of the model of the vehicle longitudinal dynamics developed By Bolender and Doman.[3,25] Because of the complexity of this model, a control-oriented model has been developed following the approach used in Ref. 16. The outputs to be controlled are the vehicle velocity and altitude. The control inputs are the elevator and canard deflections, and the fuel-to air-ratio of the scramjet engine. For this preliminary result, the presence of the flexible dynamics is not taken into account directly at the design level, but regarded as a perturbation on a nominal model, and its effect evaluated in closed-loop simulations.

The paper is organized as follows: in Section II the first-principle model is briefly introduced, and simplified curve-fitted and control-oriented models are presented; the control law design is described in Section III, and simulation results for the curve-fitted model are given in section IV. Finally, Section V concludes the paper with a brief summary of the results, and a discussion of further work.

II. Vehicle model

The vehicle model considered in this paper is the assumed-modes version[24] of the model originally developed by Bolender and Doman[3,25] for the longitudinal dynamics of a flexible air-breathing hypersonic vehicle. The equations of motions, derived using Lagrange's equations and compressible flow theory, include flexibility effects by modeling the vehicle as a single flexible structure with mass-normalized mode shapes. The scramjet engine model is taken from Chavez and Schmidt.[26] Since this first-principle model is analytically intractable, a control-oriented version of the model has been derived by replacing complex expressions of the aerodynamic forces and moments with curve-fitted approximations,[16] and by retaining the first three flexible modes. In this preliminary work, the control-oriented model is used in this paper for both control design and simulation.

A. Assumed-Modes Model

The assumed-modes model[24] considered in this paper yields the equations of motion of the vehicle longitudinal dynamics written in the stability axes as

$$
\begin{aligned}
\dot{V} &= \frac{T\cos(\alpha) - D}{m} - g\sin(\theta - \alpha) \\
\dot{h} &= V\sin(\theta - \alpha) \\
\dot{\alpha} &= -\frac{L + T\sin(\alpha)}{mV} + Q + \frac{g}{V}\cos(\theta - \alpha) \\
\dot{\theta} &= Q \\
\dot{Q} &= \frac{M}{I_{yy}} \\
\ddot{\eta}_i &= -2\zeta_i\omega_i\dot{\eta}_i - \omega_i^2\eta_i + N_i\,, \qquad i = 1,2,3
\end{aligned} \tag{1}
$$

where m is the vehicle mass, I_{yy} the moment of inertia, g the acceleration of gravity, and ζ_i and ω_i are the damping factor and the natural frequency of the i-th flexible mode, respectively. The aerodynamic forces and moments enter (1) as lift L, drag D, thrust T, pitching moment M about the body y-axis, and the generalized elastic forces N_i, $i = 1,2,3$. The model comprises five rigid-body state variables x_r, six flexible states η, and four control inputs u, as defined in Table 1:

$$x = [V, \alpha, Q, h, \theta]^{'}, \qquad \eta = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3]^{'}, \qquad u = [\Phi, A_d, \delta_e, \delta_c]^{'}$$

whereas the output to be controlled is selected as $y = [V, h]^{'}$. The equations that define the aerodynamic and generalized forces and moments (not reported here for reasons of space limitation) are quite intricate functions of the state variables, the input and the plant parameters, which do not admit a closed-form representation. The interested reader is referred to the cited references for details.

B. Curve-Fitted Model and Control-Oriented Model

The first-principle model (1) leads to equations of motion in which the relationships between control inputs and controlled outputs are not explicit. For controller design, a simplified model has been developed following the approach used in Ref. 16. The simplified model, referred to as the curve-fitted model (CFM), approximates the behavior of the assumed-modes model by replacing the aerodynamic and generalized forces and moments with curve-fitted approximations. The resulting non-linear model offers the advantage of being analytically tractable (albeit still quite complex) and more suitable for control design, while retaining the relevant dynamical features of the first-principle model. The approximations of the forces and moments employed in the CFM are given as follows

$$
\begin{aligned}
T &\approx C_{T,\Phi}(\alpha)\Phi + C_{A_d,\Phi}(\alpha)A_d + C_{T,0}(\alpha) + C_{T,\eta}\eta \\
M &\approx z_T T + \bar{q}\bar{c}SC_M(\alpha, \delta_e, \delta_c, \eta) \\
L &\approx \bar{q}SC_L(\alpha, \delta_e, \delta_c, \eta) \\
D &\approx \bar{q}SC_D(\alpha, \delta_e, \delta_c, \eta) \\
N_i &\approx N_i^{\alpha^2}\alpha^2 + N_i^{\alpha}\alpha + N_i^{\delta_e}\delta_e + N_i^{\delta_c}\delta_c + N_i^0 + N_i^{\eta}\eta
\end{aligned}
$$

where the thrust-to-moment coupling coefficient z_T and the mean aerodynamic chord $\bar{c}$ are given constants, $\bar{q}$ denotes dynamic pressure, and

$$\begin{aligned}
C_{T,\Phi}(\alpha) &= C_T^{\Phi\alpha^3}\alpha^3 + C_T^{\Phi\alpha^2}\alpha^2 + C_T^{\Phi\alpha}\alpha + C_T^{\Phi} \\
C_{T,A_d}(\alpha) &= C_T^{A_d\alpha^3}\alpha^3 + C_T^{A_d\alpha^2}\alpha^2 + C_T^{A_d\alpha}\alpha + C_T^{A_d} \\
C_{T,0} &= C_T^{\alpha^3}\alpha^3 + C_T^{\alpha^2}\alpha^2 + C_T^{\alpha}\alpha + C_T^{0} \\
C_M(\alpha,\delta_e,\delta_c,\eta) &= C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^0 + C_M^{\eta}\eta \\
C_L(\alpha,\delta_e,\delta_c,\eta) &= C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0 + C_L^{\eta}\eta \\
C_D(\alpha,\delta_e,\delta_c,\eta) &= C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^{\delta_c^2}\delta_c^2 + C_D^{\delta_c}\delta_c + C_D^0 + C_D^{\eta}\eta\,.
\end{aligned}$$

In this study, the CFM is used primarily for simulation, while for the development of the controller a control-oriented model (COM) is employed, obtained from the CFM by neglecting the flexible dynamics. As a result, the flexible dynamics are not taken into account directly at the design level, but are considered as perturbations on the COM, and their effects evaluated in simulation. Consequently, the COM comprises of the five rigid-body state variables and four control inputs. Among the available control inputs, the canard surface deflection (δ_c) will be used to decouple the lift force from the elevator, the fuel to air ratio (Φ) will be used to command thrust, and hence to control the vehicle velocity, while the elevator surface deflection will ultimately be used to control the angular dynamics through the pitch moment. Since the diffuser area ratio (A_d) will not be used as independent control input, its value will be set to $A_d = 1$ for the remainder of the paper. Furthermore, to facilitate the development of the controller, the flight-path angle $\gamma = \theta - \alpha$ will be used in place of the pitch angle as a state variable. The equations of motions for the COM are given by

$$\begin{aligned}
\dot{V} &= \frac{T\cos\alpha - D}{m} - g\sin\gamma \\
\dot{h} &= V\sin\gamma \\
\dot{\gamma} &= \frac{L + T\sin\alpha}{mV} - \frac{g}{V}\cos\gamma \\
\dot{\alpha} &= -\frac{L + T\sin\alpha}{mV} + Q + \frac{g}{V}\cos\gamma \\
\dot{Q} &= \frac{M}{I_{yy}}
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
T &\approx C_{T,\Phi}(\alpha)\Phi + \bar{C}_{T,0}(\alpha) \\
M &\approx z_T T + \bar{q}\bar{c}SC_M(\alpha,\delta_e,\delta_c,0) \\
L &\approx \bar{q}SC_L(\alpha,\delta_e,\delta_c,0) \\
D &\approx \bar{q}SC_D(\alpha,\delta_e,\delta_c,0)
\end{aligned} \tag{3}$$

and the expression of $\bar{C}_{T,0}(\alpha)$ reads as

$$\bar{C}_{T,0} = (C_T^{A_d\alpha^3} + C_T^{\alpha^3})\alpha^3 + (C_T^{A_d\alpha^2} + C_T^{\alpha^2})\alpha^2 + (C_T^{A_d\alpha} + C_T^{\alpha})\alpha + C_T^{A_d} + C_T^0\,.$$

III. Robust/Adaptive Controller Design

The control problem considered in this paper is to track given velocity and altitude references, generated using filtered step signals to account for physical limits of performance of the vehicle. The approach we consider for control design aims at avoiding writing the model in the so-called normal form for the purpose of inverting its dynamics, as traditionally done in feedback linearization schemes. The first reason behind this choice lies in the fact that the model is non-minimum phase with respect to the input $[\Phi, \delta_e]'$ and the output to be controlled $[V, h]'$. Apart from a weakly non-minimum phase behavior stemming from the flexible effects for the CFM, both the CFM and the COM possess an exponentially unstable zero-dynamics (with respect to the given I/O pair) due to the coupling between elevator deflection and lift. The occurrence of an unstable zero-dynamics renders a design based on dynamic inversion problematic, and may lead to poor closed-loop performance or even instability.[16] Secondarily, even a normal form based on the COM leads to a very complicated expression for the high-order Lie derivatives, which, together with the fact that a closed-form expression of the diffeomorphism is not available, render a robustness analysis intractable. For these reasons, we propose in this work a nonlinear design based on classic sequential loop-closure arguments, which uses a combination of adaptive and robust techniques and avoids resorting to dynamic inversion. The methodology proceeds by decomposing the equations of motions of the COM into functional subsystems, which are controlled separately using suitable virtual control inputs. For each subsystems, an appropriate control law is designed by taking into consideration the structure of the equations, the level of interaction with the rest of the dynamics, and the time scale of the variables to be controlled. For the slower portion of the dynamics (velocity and altitude), first a control law with adaptive drag compensation is derived for the velocity subsystem by controlling the thrust from Φ. Then, a controller for the altitude dynamics is developed using the flight-path angle as a virtual reference input. Finally, the fast angular dynamics with states $[\alpha, Q]'$ is used as a servo-controller (commanded by the pitch moment) to let $\gamma(t)$ track the required virtual reference command.

Since the COM is obtained from a curve-fitted approximation of a first-principle model, it is fundamental that the control law provides robustness with respect to uncertainty on the parameters of the plant model. In developing the controller, it is assumed that all the coefficients of the COM (apart from obvious ones corresponding to physically measurable quantities or known constants) are subject to a possibly large uncertainty. In particular, the design must ensure that the control objectives (in terms of boundedness of all internal variables in closed-loop and convergence of the tracking error) are maintained for all possible values of the plant parameters and for all initial conditions ranging over given compact sets.

A. Adaptive Controller for the Velocity Subsystem

Substituting the expression of the thrust T in Eq. (3) into the first equation of the COM dynamics (2), the equation

$$m\dot{V} = C_{T,\Phi}(\alpha)\cos\alpha\,\Phi + C_{T,0}(\alpha)\cos\alpha - D - mg\sin\gamma$$

is obtained for the dynamics of the vehicle velocity, in which the input Φ appears explicitly. Since, due to fuel consumption, the mass of the vehicle is uncertain and changes with time, we set $m = \tilde{m}(t)\cdot m_0$, where m_0 is the nominal mass value and $\tilde{m}(t)$ is a multiplicative uncertainty. Since $\tilde{m}(t)$ changes slowly with respect to the time scale of the references to be tracked, it is considered constant during each setpoint tracking maneuver. Defining $\tilde{V}$ as the difference between V and the reference V_{ref}, the dynamics of the tracking error for the vehicle velocity is written as

$$m\dot{\tilde{V}} = C_{T,\Phi}(\alpha)\cos\alpha\,\Phi + C_{T,0}(\alpha)\cos\alpha - D - mg\sin\gamma - m\dot{V}_{\mathrm{ref}}. \tag{4}$$

By introducing the vector of uncertain parameter

$$\begin{aligned}\theta_1 \;=\; & [C_T^{\Phi\alpha^3}, C_T^{\Phi\alpha^2}, C_T^{\Phi\alpha}, C_T^{\Phi}, C_T^{A_d\alpha^3} + C_T^{\alpha^3}, C_T^{A_d\alpha^2} + C_T^{\alpha^2}, C_T^{A_d\alpha} + C_T^{\alpha}, C_T^{A_d} + C_T^{0},\\ & SC_D^{\alpha^2}, SC_D^{\alpha}, SC_D^{\delta_e^2}, SC_D^{\delta_e}, SC_D^{\delta_c^2}, SC_D^{\delta_c}, SC_D^{0}, \tilde{m}]'\end{aligned}$$

and defining the regressor and the input matrix

$$\begin{aligned}\Psi_1'(x,u) &= [0,0,0,0,-\alpha^3\cos\alpha, -\alpha^2\cos\alpha, -\alpha\cos\alpha, -\cos\alpha,\\ &\qquad \bar{q}\alpha^2, \bar{q}\alpha, \bar{q}\delta_e^2, \bar{q}\delta_e, \bar{q}\delta_c^2, \bar{q}\delta_c, \bar{q}, m_0 g\sin\gamma - m_0\dot{V}_{\mathrm{ref}}]\\ B_1'(\alpha) &= [\alpha^3\cos\alpha, \alpha^2\cos\alpha, \alpha\cos\alpha, \cos\alpha, 0,0,0,0,0,0,0,0,0,0,0,0]\end{aligned}$$

equation (4) can be written in the linearly parameterized form

$$m\dot{\tilde{V}} \;=\; \theta_1' B_1(\alpha)\Phi - \Psi_1'(x,u)\theta_1\,. \tag{5}$$

Functional controllability of the vehicle[16] implies that $\theta_1' B_1(\alpha) \neq 0$ for all values of α within the feasible envelope of flight conditions. This conditions is assumed to hold for all θ_1 within a given compact set

$\Theta_1 \subset \mathbb{R}^{16}$ chosen to represent all possible variations of the considered plant parameters. The next step is to derive a certainty-equivalence controller using the Lyapunov function candidate $W_1(\tilde{V}) = \frac{m}{2}\tilde{V}^2$. The Lie derivative of $W_1(\tilde{V})$ along the vector field defined by the right-hand-side of Eq. (5) is given by

$$\dot{W}_1(\tilde{V}) = \tilde{V}\left(\theta_1^{'} B_1(\alpha)\Phi - \Psi_1^{'}(x,u)\theta_1\right).$$

Choosing

$$\Phi = \frac{1}{\theta_1^{'} B_1(\alpha)}\left(-k_1\tilde{V} + \Psi_1^{'}(x,u)\theta_1\right)$$

where $k_1 > 0$ is a gain parameter, the derivative of the Lyapunov function candidate is rendered negative definite. Replacing the uncertain parameter θ_1 with a suitable estimate vector $\hat{\theta}_1$, the input Φ is ultimately selected as

$$\Phi = \frac{1}{\hat{\theta}_1^{'} B_1(\alpha)}\left(-k_1\tilde{V} + \Psi_1^{'}(x,u)\hat{\theta}_1\right). \tag{6}$$

Finally, letting $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1$, the Lyapunov function derivative is written as

$$\begin{aligned}\dot{W}_1 &= \tilde{V}\,(\hat{\theta}_1^{'} - \tilde{\theta}_1^{'})B_1(\alpha)\Phi - \tilde{V}\,\Psi_1^{'}(x,u)\big(\hat{\theta}_1 - \tilde{\theta}_1\big)\\ &= -k_1\tilde{V}^2 + \tilde{\theta}_1^{'}\big(\tilde{V}\,\Psi_1(x,u) - \tilde{V}\,B_1(\alpha)\Phi\big)\end{aligned} \tag{7}$$

The choice of the update law for the parameter estimate $\hat{\theta}_1$, aiming at canceling the sign-indefinite terms in Eq. (7), will be postponed till later in this section. Note that to ensure non-singularity of the control law (6) over the envelope of feasible flight conditions, it suffices to constrain the estimates $\hat{\theta}_1(t)$ to evolve within a suitable compact convex set which includes the compact set Θ_1. This can be easily accomplished by resorting to smooth parameter projection methods in the definition of the update law.[21]

B. Robust Integral Controller for the Altitude Subsystem

Given a desired reference profile h_{ref} for the altitude, the corresponding tracking error is defined as $\tilde{h} = h - h_{\mathrm{ref}}$. Since for small values of the flight path angle, $\dot{h} = V\sin\gamma \approx V\gamma$, it follows that

$$\dot{\tilde{h}} = \dot{h} - \dot{h}_{\mathrm{ref}} \approx \gamma V - \dot{h}_{\mathrm{ref}}.$$

Choosing a reference command for the flight-path angle as

$$\gamma_{\mathrm{ref}} = \frac{-k_h\big(h - h_{\mathrm{ref}}\big) + \dot{h}_{\mathrm{ref}}}{V}, \tag{8}$$

where $k_h > 0$ is a gain parameter, the corresponding dynamics for the altitude tracking error satisfies

$$\dot{\tilde{h}} = -k_h\tilde{h}$$

and thus, if the flight-path angle is controlled to follow the command γ_{ref}, the altitude tracking error is regulated to zero exponentially fast. Note that the reference trajectory (8) is well-defined, as $V > 0$ in any feasible flight conditions. Next, we proceed to design a control law to let $\gamma(t)$ track $\gamma_{\mathrm{ref}}(t)$ asymptotically, using the angle of attack as a virtual control input.

In controlling the flight-path angle dynamics of the vehicle, one of the major obstacle is its non-minimum phase characteristic, due to the explicit dependence of the lift force on the control surfaces.[27] In order to eliminate this non-minimum phase behavior, the canard is used to suppress the influence of the elevator on the lift force by ganging the canard deflection to that of the elevator. Specifically, the choice

$$\delta_c = -\frac{C_L^{\delta_e}}{C_L^{\delta_c}}\delta_e \tag{9}$$

achieves direct cancelation of the elevator-to-lift contribution. To account for parameter uncertainty, the selection of the coupling coefficient between the canard and the elevator deflection can be endowed with a self-tuning mechanism. In this preliminary result, for the sake of simplicity, the choice of δ_c will be kept fixed as in (9). After compensation, the lift equation reads as

$$L = \bar{q}S\big(C_L^\alpha \alpha + C_L^0\big)\,. \tag{10}$$

Using Eq. (10) and the first equation in (3), the dynamics of the flight-path angle can be written in the form

$$mV\,\dot{\gamma} = \bar{q}S\left(C_L^\alpha \alpha + C_L^0\right) + C_{T,\Phi}(\alpha)\sin\alpha\,\Phi + C_{T,0}(\alpha)\sin\alpha - mg\,\cos\gamma \tag{11}$$

Defining the tracking error for the flight-path angle as $\tilde{\gamma} = \gamma - \gamma_{\text{ref}}$, it follows that

$$\begin{aligned}\dot{\tilde{\gamma}} &= \frac{1}{mV}\left(\bar{q}S\left(C_L^\alpha \alpha + C_L^0\right) + C_{T,\Phi}(\alpha)\sin\alpha\,\Phi + C_{T,0}(\alpha)\sin\alpha - mg\,\cos(\tilde{\gamma}+\gamma_{\text{ref}}) - mV\dot{\gamma}_{\text{ref}}\right)\\ &:= f(\alpha,\tilde{\gamma},\bar{q},\Phi,\gamma_{\text{ref}})\,.\end{aligned} \tag{12}$$

The velocity and altitude reference trajectories bring the vehicle to a desired trim condition, defined as

$$x^* = [V^*, h^*, \alpha^*, \gamma^*, Q^*]', \qquad u^* = [\Phi^*, \delta_e^*, \delta_c^*]'$$

where, in particular,

$$V^* = \lim_{t\to\infty} V_{\text{ref}}(t), \qquad h^* = \lim_{t\to\infty} h_{\text{ref}}(t), \qquad \gamma^* = \lim_{t\to\infty} \gamma_{\text{ref}}(t) = 0, \qquad Q^* = 0$$

and the trim condition for Φ is determined by the steady-state value of the parameter estimates $\hat{\theta}_1^* = \lim_{t\to\infty}\hat{\theta}_1(t)$

$$\Phi^* = \frac{1}{\hat{\theta}_1^{*\prime} B_1(\alpha^*)}\Psi_1'(x^*,u^*)\,\hat{\theta}_1^*\,. \tag{13}$$

Furthermore, we let $\bar{q}$ and α^* denote respectively the value of the dynamic pressure and the angle of attack at the desired trim condition. Expanding Eq. (12) around α^*, yields

$$\dot{\tilde{\gamma}} = f_0(\alpha^*,\tilde{\gamma},\bar{q},\Phi,\gamma_{\text{ref}}) + f_1(\alpha^*,\Phi)(\alpha-\alpha^*) + R(\alpha-\alpha^*) \tag{14}$$

where

$$f_1(\alpha^*,\Phi) = \frac{1}{mV}\Big[\,\bar{q}S\,C_L^\alpha + \frac{\partial(C_{T,\Phi}(\alpha)\sin\alpha)}{\partial\alpha}\Big|_{\alpha=\alpha^*}\Phi + \frac{\partial(C_{T,0}(\alpha)\sin\alpha)}{\partial\alpha}\Big|_{\alpha=\alpha^*}\Big]$$

and $R(\alpha-\alpha^*)$ is a function vanishing at $\alpha=\alpha^*$ together with its first derivative. To obtain an equilibrium at trim, the following condition needs to be satisfied

$$f_0(\alpha^*, 0, \bar{q}^*, \Phi^*, 0) = 0\,. \tag{15}$$

Solving (15) yields the trim condition α^* that enforces the equilibrium in $\tilde{\gamma}=0$. In particular, it is seen that

$$\begin{aligned}f_0(\alpha^*,\tilde{\gamma},\bar{q},\Phi,\gamma_{\text{ref}}) &= (\bar{q}-\bar{q}^*)S(C_L^\alpha\alpha^* + C_L^0) + C_{T,\Phi}(\alpha^*)\sin\alpha^*(\Phi-\Phi^*) - mV\dot{\gamma}_{\text{ref}}\\ &\quad -mg\,\cos(\gamma_{\text{ref}}) + mg\left[(1-\cos\tilde{\gamma})\,\cos(\gamma_{\text{ref}}) + \sin\tilde{\gamma}\,\sin(\gamma_{\text{ref}})\right]\end{aligned}$$

which vanishes at trim. The value at trim for the angle of attack must necessarily be generated by the controller to maintain the desired trim condition in closed-loop. However, it is not possible to compute α^* analytically using (15), since f_0 depends on uncertain plant parameters. To overcome this problem, the system is augmented with an integrator, whose state has the purpose of reconstructing asymptotically α^* in closed-loop. The integrator dynamics is given as

$$\dot{\xi} = v \tag{16}$$

where the input $v = v(\tilde{\gamma})$ is chosen as a function of $\tilde{\gamma}$ which vanishes at $\tilde{\gamma}=0$. Letting $\tilde{\xi} = \xi - \alpha^*$, the interconnection of system (14) with system (16) reads as

$$\begin{aligned}\dot{\tilde{\gamma}} &= f_1(\alpha^*,\Phi)(\alpha-\alpha^*) + f_0(\alpha^*,\tilde{\gamma},\bar{q},\Phi,\gamma_{\text{ref}}) + R(\alpha-\alpha^*)\\ \dot{\tilde{\xi}} &= v(\tilde{\gamma})\end{aligned} \tag{17}$$

where the angle of attack (which plays the role of a virtual control input for system (17)) and the function $v(\tilde{\gamma})$ must be chosen in such a way that the equilibrium $[\tilde{\gamma}, \tilde{\xi}]' = [0, 0]'$ is asymptotically stable. The desired angle of attack is chosen as

$$\alpha_d = \xi - k_2 \tilde{\gamma} \,, \tag{18}$$

where $k_2 > 0$ is a gain parameter. Assuming $\xi(t) \to \alpha^*$, the first term in the right-hand side of equation (18) enforces the equilibrium at trim, whereas the second term has a stabilizing effect, since $f_1(\alpha^*, \Phi) \gg 0$ in the range of plant parameters and state variables of interest. Therefore, imposing (18) yields

$$\alpha - \alpha^* = \tilde{\xi} - k_2 \tilde{\gamma} + \tilde{\alpha} \,, \tag{19}$$

where $\tilde{\alpha}$ denotes the tracking error for the desired dynamic of the angle of attack. The control $v(\tilde{\gamma})$ is chosen simply as

$$v(\tilde{\gamma}) = -k_2 \tilde{\gamma} \,.$$

To proceed with the analysis, we set $\tilde{\alpha} = 0$ and verify if there exists a choice of the gain k_2 for which the system

$$\begin{aligned} \dot{\tilde{\gamma}} &= f_0(\alpha^*, \tilde{\gamma}, \bar{q}, \Phi, \gamma_{\text{ref}}) + f_1(\alpha^*, \Phi)(\tilde{\xi} - k_2 \tilde{\gamma}) + R(\tilde{\xi} - k_2 \tilde{\gamma}) \\ \dot{\tilde{\xi}} &= v(\tilde{\gamma}) \end{aligned} \tag{20}$$

is asymptotically stable. To this purpose, the following change of coordinates

$$\chi = \tilde{\xi} - \frac{1}{f_1(\alpha^*, \Phi)} \, \tilde{\gamma} \tag{21}$$

is applied to system (20). Since the velocity dynamics are slower than the angular dynamics, $\Phi(t)$ can be considered constant in the time scale of system (20). As a result, it is possible to consider the following approximation for the dynamics of the integral error in the new coordinates χ:

$$\dot{\chi} \approx -k_2 \tilde{\gamma} - \frac{1}{f_1(\alpha^*, \Phi)} \, \dot{\tilde{\gamma}} \,.$$

For the sake of notational convenience, in what follows we suppress the arguments from the functions $f_0(\alpha^*, \tilde{\gamma}, \bar{q}, \Phi, \gamma_{\text{ref}})$ and $f_1(\alpha^*, \Phi)$. In the new coordinates, the system (20) assumes the form

$$\begin{aligned} \dot{\chi} &= -\chi - k_2 \tilde{\gamma} + (k_2 - \frac{1}{f_1}) \, \tilde{\gamma} - \frac{f_0}{f_1} - \frac{1}{f_1} R(\chi - (k_2 - \frac{1}{f_1}) \, \tilde{\gamma}) \\ \dot{\tilde{\gamma}} &= f_0 - (f_1 k_2 - 1) \, \tilde{\gamma} + f_1 \chi + R(\chi - (k_2 - \frac{1}{f_1}) \, \tilde{\gamma}) \,. \end{aligned} \tag{22}$$

Looking at the linear approximation of the system (22) around the origin, which reads as

$$\begin{aligned} \dot{\chi} &= -\chi + [-\frac{1}{f_1} - \frac{1}{f_1} mg \, \sin(\gamma_{\text{ref}})] \, \tilde{\gamma} \\ \dot{\tilde{\gamma}} &= -[f_1 k_2 - 1 - mg \, \sin(\gamma_{\text{ref}})] \, \tilde{\gamma} + f_1 \chi \,, \end{aligned} \tag{23}$$

it becomes clear that, when $\tilde{\gamma} = 0$, the χ-subsystem in Eq.23 is exponentially stable, and when $\chi = 0$ the $\tilde{\gamma}$-subsystem is exponentially stable for k_2 large enough (recall that $f_1(\alpha^*, \Phi) \gg 0$). As a result, each subsystem is input-to-state stable[22] (ISS), with linear gains given by

$$\begin{aligned} \gamma_{12} &:= \sup_{t \geq 0} \left| -\frac{1}{f_1} - \frac{1}{f_1} mg \, \sin(\gamma_{\text{ref}}(t)) \right| \\ \gamma_{21} &:= \sup_{t \geq 0} \left| \frac{f_1}{f_1 k_2 - 1 - mg \, \sin(\gamma_{\text{ref}}(t))} \right| \,. \end{aligned}$$

The gain of the overall interconnection is given by the product of the single gains, from which it is evident that there exists a value $k_2^* > 0$ such that $\gamma_{12}\gamma_{21} < 1$ for all $k_2 \geq k_2^*$. As a consequence, by the small-gain theorem, the system (23) is globally exponentially stable for $\forall k_2 \geq k_2^*$. As a consequence, the system (22) is locally exponentially stable as well. It should be noted that, while the linear system (23) possesses an infinite gain margin with respect to k_2, the original nonlinear system (22) need not share the same property, due to the appearance of k_2 in the higher-order term $R(\tilde{\xi} - k_2\tilde{\gamma})$. As a consequence, the choice of the gain k_2 must be made to achieve the largest possible domain of attraction for the origin of system (22), for all possible values of the plant parameters. This can be accomplished using numerical optimization methods by ensuring that that the Lie derivative of the Lyapunov function candidate $W_2(\tilde{\gamma}, \chi) = \frac{1}{2}\big(\tilde{\gamma}^2 + \chi^2\big)$, evaluated along the vector field of system (22), is negative definite in a level set $\Omega_c = \{W_2(\tilde{\gamma}, \chi) \leq c\}$ which contains the required envelope on initial conditions, for all the considered range of the values of the plant parameters. Specifically, letting $z = [\tilde{\gamma}, \chi]'$ for notational convenience, and substituting (19) in equation (14), the $(\tilde{\gamma}, \chi)$-dynamics can be written in the form

$$\dot{z} = \varphi_1(z) + \tilde{\varphi}_1(z, \tilde{\alpha})\, \tilde{\alpha}\,. \tag{24}$$

It is assumed that k_2 has been selected such that the system $\dot{z} = \varphi_1(z)$ has a locally exponentially stable equilibrium at $z = 0$, and satisfies the uniform Lyapunov stability property[23] with respect to the Lyapunov function $W_2(z) = \frac{1}{2}\|z\|^2$. In particular, it is assumed that there exists a constant $c_1 \geq 1$ such that the set $\Omega_{c_1} = \{z : W_2(z) \leq c_1 + 1\}$ is compact, contains all the initial conditions of interest and is such that, for all values of the uncertain plant parameters,

$$\frac{\partial W_2}{\partial z}\, \varphi_1(z) \leq -\Upsilon_1(z) \qquad \forall\, z \in \Omega_{c_1} \tag{25}$$

where $\Upsilon_1(z)$ is a continuous and positive definite function. Following the approach considered in Ref. 23, the Lyapunov function $W_2(z)$ is modified as

$$\tilde{W}_2(z) = c_1\, \frac{W_2(z)}{c_1 + 1 - W_2(z)}$$

so that $\tilde{W}_2(z)$ is proper on the set Ω_{c_1} while at the same time satisfying

$$\frac{\partial \tilde{W}_2}{\partial z}\, \varphi_1(z, 0) \leq -\tilde{\Upsilon}_1(z) \qquad \forall z \in \Omega_{c_1}$$

for some continuous and positive definite function $\tilde{\Upsilon}_1(z)$. Moreover, since the stability is exponential, there exists $a > 0$ and $r > 0$ such that $\tilde{\Upsilon}_1(z) \geq a\|z\|^2$ for all z such that $\|z\| < r$.

C. Robust Controller for the $(\tilde{\alpha}, Q)$-Subsystem

Next, we consider the problem of controlling the angular dynamics to regulate asymptotically $\tilde{\alpha}(t)$ to zero. For this purpose, the control input for the angular dynamics given by the (α, Q)-subsystem in Eq. 2 is initially selected as the pitch moment M. The approach we take for controller design is based on robust semi-global stabilization methods.[19,23]

Let $k_3 > 0$ be a gain parameter, and consider the change of coordinates

$$[\alpha,\, Q]' \rightarrow \zeta = [\zeta_1,\, \zeta_2]' := \left[\tilde{\alpha},\, \tfrac{1}{k_3} Q\right]' \tag{26}$$

yielding a system of the form

$$\dot{\zeta} = k_3 F_0 \zeta + G_1 \varphi_2(\zeta, z) + G_2 M\,, \tag{27}$$

where

$$F_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad G_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 0 \\ \frac{1}{I_{yy} k_3} \end{pmatrix}$$

and $\varphi_2(\zeta, z)$ is a function vanishing at the origin. The desired value for the pitching moment is selected as

$$\begin{aligned} M_d &= -k_3^2\, a_0\, \tilde{\alpha} - k_3\, a_1\, Q && (28) \\ &= k_3^2\big(- a_0\, \zeta_1 - a_1\, \zeta_2\big) && (29) \end{aligned}$$

where a_0 and a_1 are such that $p(\lambda) = \lambda^2 + a_1\lambda + a_0$ is a Hurwitz polynomial, the system (27) can be rewritten in the following form:

$$\dot{\zeta} = k_3 F\zeta + G_1\varphi_2(\zeta, z) + G_2(M - M_d) \tag{30}$$

where

$$F = \begin{pmatrix} 0 & 1 \\ -\dfrac{a_0}{I_{yy}} & -\dfrac{a_1}{I_{yy}} \end{pmatrix}.$$

Since by construction F is a Hurwitz matrix for all $I_{yy} > 0$, for any positive numbers $I_1 < I_2$ there exists a fixed symmetric and positive definite matrix P that satisfies the Lyapunov equation $F^{'}P + PF \leq -I$ for all $I_{yy} \in [I_1, I_2]$. Applying the change of coordinates (26) to the equation (24), it is possible to group the z- and ζ-system equations as

$$\begin{aligned} \dot{z} &= \varphi_1(z) + \tilde{\varphi}_1(z, \zeta_1)\,\zeta_1 & (31)\\ \dot{\zeta} &= k_3 F\zeta + G_1\varphi_2(\zeta, z) + G_2(M - M_d) & (32) \end{aligned}$$

and consider the overall Lyapunov function $W_3(z, \zeta) = \tilde{W}_2(z) + \zeta^{'}P\zeta$. Since $\tilde{W}_2$ is positive definite, $\frac{\partial \tilde{W}_2}{\partial z}|_{z=0} = 0$ and therefore $\frac{\partial \tilde{W}_2}{\partial z} = \tilde{v}_2(z)\,z$ for some continuous function $\tilde{v}_2(z)$. Moreover, since $\varphi_2(0, 0) = 0$, it is possible to write

$$\varphi_2(\zeta, z) = \varphi_{2,1}(\zeta, z)\,\zeta + \varphi_{2,2}(\zeta, z)\,z$$

for some continuous functions $\varphi_{2,1}(\zeta, z)$ and $\varphi_{2,2}(\zeta, z)$. Due to the fact that the function $W_3(z, \zeta)$ is proper on $\Omega_{c_1} \times \mathbb{R}^2$, there exist constants $c_2 > 0$ and $c_3 > 0$ such that the set $\mho = \{(z, \zeta) : c_2 \leq W_3(z, \zeta) \leq c_3\}$ is compact, and the level sets $\Omega_{c_2} = \{(z, \zeta) : W_3(z, \zeta) \leq c_2\}$ and $\Omega_{c_3} = \{(z, \zeta) : W_3(z, \zeta) \leq c_3\}$ are such that $\Omega_{c_2} \subset \mathcal{B}_r$, and Ω_{c_3} contains all initial conditions of interest. Defining

$$\begin{aligned} \mu_1 &:= \max_{(z,\zeta)\in\mho} \|\tilde{v}_2(z)\,\tilde{\varphi}_1(z, \zeta_1)\| \\ \mu_2 &:= 2\|P\|\,\|G_1\| \max_{(z,\zeta)\in\mho} \|\varphi_{2,1}(\zeta, z)\| \\ \mu_3 &:= 2\|P\|\,\|G_2\| \max_{(z,\zeta)\in\mho} \|\varphi_{2,2}(\zeta, z)\| \end{aligned}$$

it follows that the derivative of $W_3(z, \zeta)$ along the trajectories of the system (31) satisfies

$$\begin{aligned} \dot{W}_3 &= \frac{\partial W_3}{\partial z}\,\varphi_1(z) + \frac{\partial W_3}{\partial z}\,\tilde{\varphi}_1(z, \zeta_1)\,\zeta_1 + \frac{\partial W_3}{\partial \zeta}\,[k_3F\zeta + G_1\varphi_2(\zeta, z)] + \frac{\partial W_3}{\partial \zeta}\,G_2(M - M_d) \\ &\leq -\tilde{\Upsilon}_1(z) - k_3\|\zeta\|^2 + \frac{\partial \tilde{W}_2}{\partial z}\,\tilde{\varphi}_1(z, \zeta_1)\,\zeta_1 + 2\zeta^{'}PG_1\varphi_2(\zeta, z) + 2\zeta^{'}PG_2(M - M_d) \\ &\leq -\tilde{\Upsilon}_1(z) - k_3|\zeta|^2 + \mu_1|z||\zeta| + \mu_2|\zeta|^2 + \mu_3|z||\zeta| + 2\zeta^{'}PG_2(M - M_d) \end{aligned}$$

for all $(z, \zeta)\ in\Omega_{c_3}$, and for all values of the plant parameters in a given compact set. As a result, there exists $k_3^* > 0$ such that for all $k_3 > k_3^*$ the trajectories of system (31) when $M = M_d$ are captured by the set Ω_{c_2}. Finally, since the set Ω_{c_2} is contained in the domain of exponential stability, it follows that the origin is an asymptotically stable equilibrium of the system (31), with domain of attraction which includes the level set Ω_{c_3}. In particular, it can be shown that there exists a function $\Upsilon_2(z, \zeta)$ which is continuous and positive definite on Ω_{c_3}, such that

$$\dot{W}_3 \leq -\Upsilon_2(z, \zeta) + 2\zeta^{'}PG_2(M - M_d) \tag{33}$$

for all $(z, \zeta) \in \Omega_{c_3}$.

D. Adaptive Controller for the Pitch Moment

The final step in our design is to determine the control law for δ_e in such a way that $|M(t) - M_d(t)| \to 0$. To this end, let us rewrite the thrust and moment expressions in matrix form. In particular, let

$$T = \theta_1^{'}B_T(\alpha)\Phi - \Psi_T^{'}(\alpha)\theta_1$$

where

$$\begin{aligned} B_T'(\alpha) &= [\alpha^3, \alpha^2, \alpha, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \\ \Psi_T'(\alpha) &= [0, 0, 0, 0, -\alpha^3, -\alpha^2, -\alpha, -1, 0, 0, 0, 0, 0, 0, 0, 0] \,. \end{aligned}$$

Recall the expression of the pitch moment

$$M = z_T T + \bar{q}\, S\, \bar{c}\, [C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^0], .$$

Since δ_c has been chosen as in (9), by defining

$$C_M^{\delta} := C_M^{\delta_e} - C_M^{\delta_c}\frac{C_L^{\delta_e}}{C_L^{\delta_c}} \,,$$

it is possible to derive the following expression for M

$$M = z_T(\theta_1' B_T \Phi - \Psi_T' \theta_1) + \theta_2' B_2 \delta_e - \Psi_2' \theta_2$$

where

$$\begin{aligned} \theta_2' &= [S\,\bar{c}\,C_M^{\alpha^2}, S\,\bar{c}\,C_M^{\alpha}, S\,\bar{c}\,C_M^{\delta}, S\,\bar{c}\,C_M^{0}] \\ B_2'(x) &= [0, 0, \bar{q}, 0] \\ \Psi_2'(x) &= [\bar{q}\alpha^2, \bar{q}\alpha, 0, \bar{q}] \,. \end{aligned}$$

The choice of the update law for the parameter estimates, including those employed for the velocity controller earlier in the Section, is obtained looking at the derivative of the Lyapunov function candidate

$$W_4(\tilde{V}, z, \zeta) = W_1(\tilde{V}) + W_3(z, \zeta)$$

along trajectories of the system defined by equations (5) and (31) can be obtained using equations (7) and (33) as

$$\begin{aligned} \dot{W}_4 &\le -k_1\tilde{V}^2 + \hat{\theta}_1'[\tilde{V}\ \Psi_1 - \tilde{V}\ B_1\Phi] - \Upsilon_2(z,\zeta) + 2\zeta' P G_2 (M - M_d) \\ &\le -k_1\tilde{V}^2 - \Upsilon_2(z,\zeta) + \hat{\theta}_1'[\tilde{V}\ \Psi_1 - \tilde{V}\ B_1\Phi] + 2\zeta' P G_2 [z_T(\theta_1' B_T\Phi - \Psi_T'\theta_1) + \theta_2' B_2 \delta_e - \Psi_2'\theta_2 - M_d] \\ &\le -k_1\tilde{V}^2 - \Upsilon_2(z,\zeta) + \hat{\theta}_1'[\tilde{V}\ \Psi_1 - \tilde{V}\ B_1\Phi] + 2\zeta' P G_2 [(\Phi z_T B_T' - z_T \Psi_T')\theta_1 - \Psi_2'\theta_2 + \theta_2' B_2\delta_e - M_d] \,. \end{aligned}$$

Letting

$$\theta = [\theta_1', \theta_2']' \,, \qquad B_3 = [0, B_2']', \qquad \Psi_3 = [-\Phi z_T B_T' + z_T \Psi_T', \Psi_2']' \ :=\ [\Psi_{3,1}', \Psi_{3,2}']'$$

then

$$\dot{W}_4 \le -k_1\tilde{V}^2 - \Upsilon_2(z,\zeta) + \hat{\theta}_1'[\tilde{V}\ \Psi_1 - \tilde{V}\ B_1\Phi] + 2\zeta' P G_2 [\theta' B_3 \delta_e - \Psi_3'\theta - M_d] \,.$$

Choosing

$$\delta_e = \frac{1}{\hat{\theta}' B_3}[\Psi_3'\theta + M_d]$$

where $\hat{\theta}$ is an estimate of θ, it follows that

$$\begin{aligned} \theta' B_3 \delta_e - \Psi_3'\theta - M_d &= (\hat{\theta}' - \tilde{\theta}')B_3\delta_e - \Psi_3'(\hat{\theta} - \tilde{\theta}) - M_d \\ &= -\tilde{\theta}' B_3 \delta_e + \Psi_3'\tilde{\theta} \\ &= -\tilde{\theta}_2' B_2 \delta_e + \Psi_{3,1}'\tilde{\theta}_1 + \Psi_{3,2}'\tilde{\theta}_2 \end{aligned}$$

where $\tilde{\theta} = \hat{\theta} - \theta$. Therefore, the derivative of the Lyapunov function candidate W_4 satisfies

$$\dot{W}_4 \le -k_1\tilde{V}^2 - \Upsilon_2(z,\zeta) + \tilde{\theta}_1'[\tilde{V}\ \Psi_1 - \tilde{V}\ B_1\Phi + 2\Psi_{3,1} G_2' P\zeta] + \tilde{\theta}_2'(\Psi_{3,2} - B_2\delta_e) G_2' P \zeta \,.$$

Table 2. Trim conditions considered in the simulation case study

Variable	Initial trim condition	Desired trim condition
V	7846.36 ft/s	8274.96 ft/s
h	85000 ft	96734.4 ft
α	0.0416 rad	0.0619 rad

Finally, consider the overall Lyapunov function

$$W(\tilde{V}, z, \zeta, \tilde{\theta}_1, \tilde{\theta}_2) = W_4(\tilde{V}, z, \zeta) + \tilde{\theta}_1' \Gamma_1^{-1} \tilde{\theta}_1 + \tilde{\theta}_2' \Gamma_2^{-1} \tilde{\theta}_2$$

which accounts for parameter estimate errors weighted by positive definite matrices Γ_1 and Γ_2. It is easy to check that the time derivative of W along the trajectories of the system satisfies

$$\begin{aligned} \dot{W} &= \dot{W}_4 + \tilde{\theta}_1' \Gamma_1^{-1} \dot{\hat{\theta}}_1 + \tilde{\theta}_2' \Gamma_2^{-1} \dot{\hat{\theta}}_2 \\ &\leq -k_1 \tilde{V}^2 - \Upsilon_2(z, \zeta) + \tilde{\theta}_1' [\tilde{V}\ \Psi_1 - \tilde{V}\ B_1 \Phi + 2\Psi_{3,1} G_2' P \zeta + \Gamma_1^{-1} \dot{\hat{\theta}}_1] + \tilde{\theta}_2' [(\Psi_{3,2} - B_2 \delta_e) G_2' P \zeta + \Gamma_2^{-1} \dot{\hat{\theta}}_2] \,. \end{aligned}$$

By choosing the update laws as

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \Gamma_1 [\tilde{V}\ (B_1 \Phi - \Psi_1) - 2\Psi_{3,1} G_2' P \zeta] \\ \dot{\hat{\theta}}_2 &= \Gamma_2 [(B_2 \delta_e - \Psi_{3,2}) G_2' P \zeta] \end{aligned}$$

one obtains

$$\dot{W} \leq -k_1 \tilde{V}^2 - \Upsilon_2(z, \zeta) \,.$$

Standard arguments[28] imply that all trajectories of the closed-loop system are bounded, and that the tracking errors $\tilde{V}(t)$ and $\tilde{\gamma}(t)$ are regulated to zero asymptotically.

IV. Simulation Results

To validate the controller derived in the previous section, several simulations have been performed on the CFM model implemented in SIMULINK®. As a representative case study, the vehicle is initially at the trim condition given in the first column of Table 2. The reference trajectory brings the vehicle to the new trim condition shown in the second column of Table 2. The reference commands have been generated by filtering step increments in velocity and altitude by a second-order pre-filter with natural frequency $\omega_f = 0.03$ rad/s and damping factor $\zeta_f = 0.9$. The tracking reference for the flight path angle is generated using equation (8), where the gain has been set equal to $k_h = 0.01$. The initial conditions of the plant parameter estimates have been randomly selected within 40% of their nominal values. Finally, the controller gains are reported in Table 3. Figure (1) shows that the tracking performance in closed-loop for the velocity, flight-path angle, and altitude, respectively. It can be seen that the tracking error for the flight-path angle converges on a faster time scale with respect to the other variable, most noticeable the altitude. The tracking performance is quite good, in spite of parameter uncertainty, and the presence of the flexible dynamics, neglected in the controller design. The flexible states remain well-behaved, as shown in Figures 2(a)-(b). Figures 2(c)-(d) show respectively the time-history of the state of the integral augmentation of the flight-path angle dynamics and two selected parameter estimates, specifically those for the stability derivative $\theta_{10} = SC_D^{\alpha}$ and the coefficient $\theta_{15} = SC_D^0$. In particular, while the estimates $\hat{\theta}(t)$ do not converge to their true values, as the reference trajectory is not sufficiently rich to ensure persistence of excitation of the regressor, the estimates are well-behaved, and settle to constant values. On the other hand, the steady-state value of the integrator converges to the trim condition α^* given in the last row of Table 2. Finally, Figures 2(d)-(e) show respectively thrust and pitch moment, and the main control inputs. The choice of a relatively low-bandwidth prefilter and a careful selections of the controller gains help maintaining both $\Phi(t)$ and $\delta_e(t)$ remain within feasible limits.

Table 3. **Design parameters and gains value**

Gain	Value	Gain	Value
k_h	0.01	a_0	5
k_1	80	a_1	15
k_2	100	Γ_1	$10 \times I_{16\times 16}$
k_3	4	Γ_2	$10 \times I_{4\times 4}$

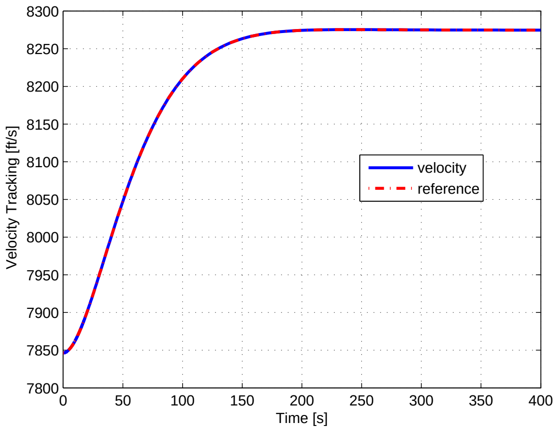
V. Conclusion

In this paper we have presented the design of a robust and adaptive nonlinear controller for the longitudinal motion of a air-breathing hypersonic vehicle. In an attempt to design controllers which are robust to model uncertainty, the methodology proposed in this paper departs from the use of dynamic inversion, in favor of an approach based on sequential loop-closure. To this end, a variety of techniques ranging from adaptive control to robust semi-global gain assignment, has been utilized in the definition of appropriate intermediate control laws at each step of the design. Simulation results reported for a specific case study show the effectiveness of the methodology.

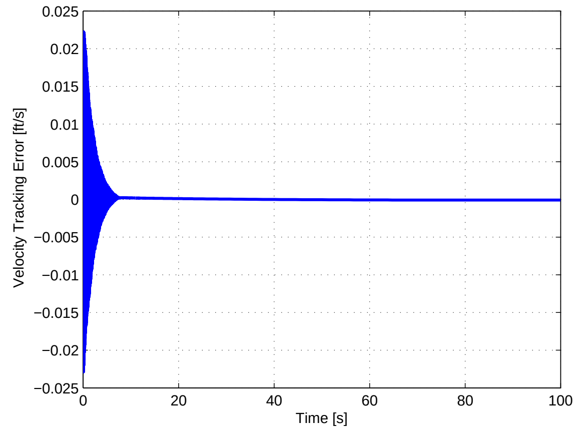
Despite being encouraging in many ways, the results presented here are to be considered preliminary, and point to much further work ahead. First of all, the presence of flexible effects must be taken into account directly at the level of control design, or at least their effect on closed-loop performance evaluated quantitatively. Second, extensive simulations should be performed on the basis of the original high-fidelity first-principle model, to validate the appropriateness of the curve-fitted model for this type of design. Since the adaptive controller tends to have a relatively high dimensionality, issues of controller order reduction should be addressed to obtain more agile controllers. Finally, the important issues of limitation in the control authority should be addressed directly at the level of controller design, to allow more aggressive maneuvers, and increase the speed or response to set-point commands.

Acknowledgments

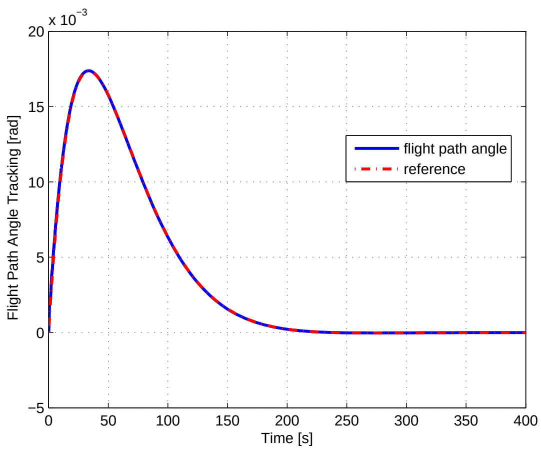
This study has been supported by the AFRL/AFOSR Collaborative Center of Control Science at the Ohio State University.



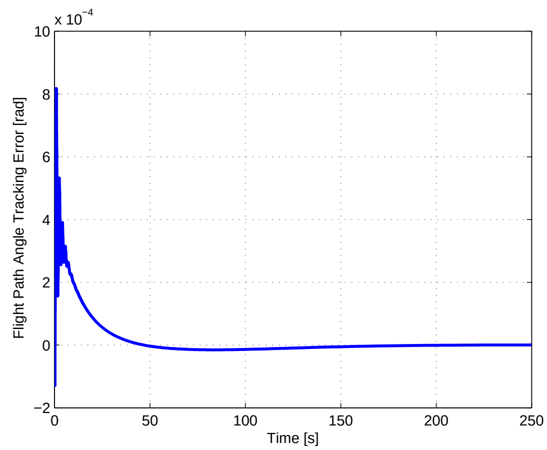
(a) Velocity Tracking



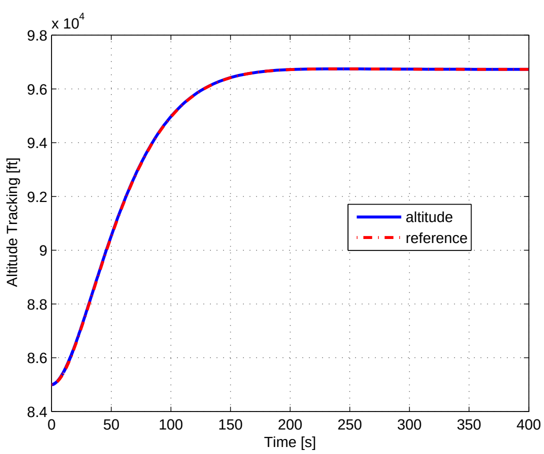
(b) Velocity Tracking Error



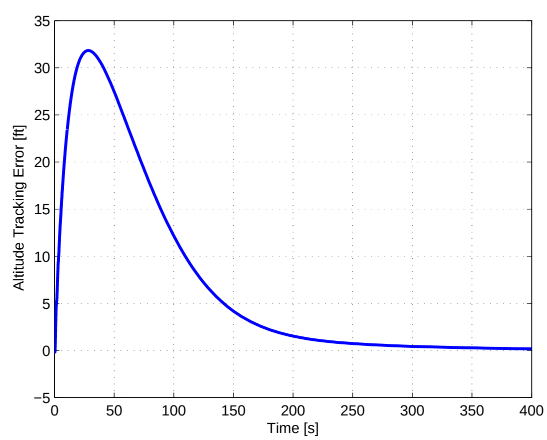
(c) Flight-Path Angle Tracking



(d) Flight-Path Angle Tracking Error

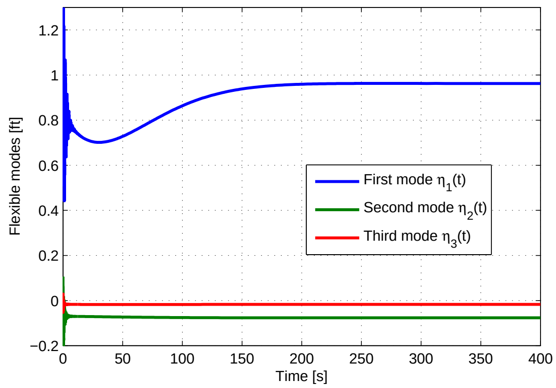


(e) Altitude Tracking

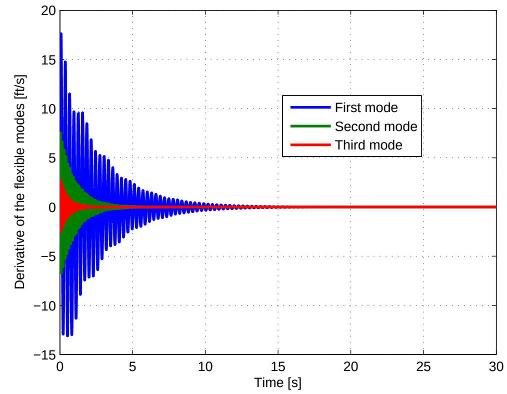


(f) Altitude Tracking Error

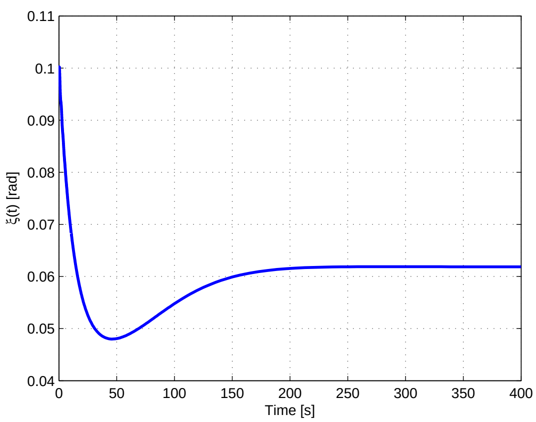
Figure 1. Simulation Results for the Control-Oriented Model: Controlled Outputs.



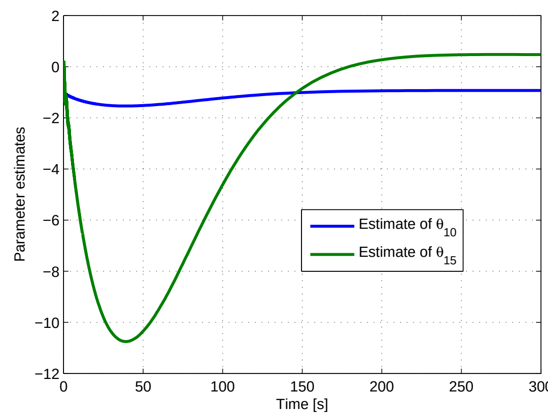
(a) Flexible States $\eta_i(t)$



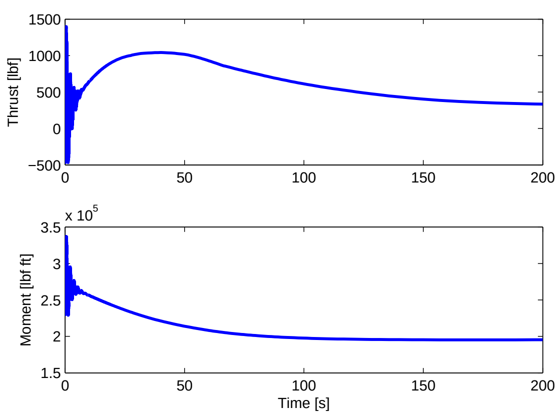
(b) Time-Derivative of the Flexible States, $\dot{\eta}_i(t)$



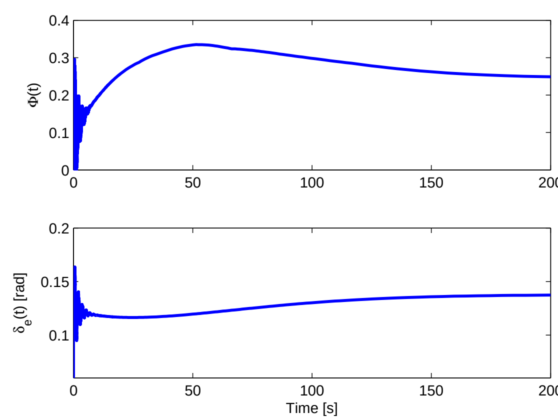
(c) Integrator State $\xi(t)$



(d) Estimates of $\theta_{10} = SC_D^{\alpha}$ and $\theta_{15} = SC_D^{0}$



(e) Thrust $T(t)$ (top) and Moment $M(t)$ (bottom)



(f) Control Inputs $\Phi(t)$ (top) and $\delta_e(t)$ (bottom)

Figure 2. Simulation Results for the Control-Oriented Model: Flexible States, Parameter Estimates, and Control Inputs.

References

[1]Schmidt, D. K., "Dynamics and Control of Hypersonic Aeropropulsive/Aeroelastic Vehicles," AIAA Paper 92-4326, Aug. 1992.

[2]Schmidt, D. K., "Integrated Control of Hypersonic Vehicles - A Necessity Not Just a Possibility," AIAA Paper 93-3761, 1993.

[3]Bolender, M. A. and Doman, D. B., "A Nonlinear Longitudinal Dynamical Model of an Air-breathing Hypersonic Vehicle," *Submitted to AIAA Journal of Spacecraft and Rockets*, 2006.

[4]McRuer, D., "Design and Modeling Issues for Integrated Airframe/Propulsion Control of Hypersonic Flight Vehicles," American Control Conference, Boston, MA, 1992, pp. 729–734.

[5]Bilimoria, K. and Schmidt, D., "Integrated Development of the Equations of Motion for Elastic Hypersonic Flight Vehicles," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 1, 1995, pp. 73–81.

[6]Schmidt, D., "Optimum mission performance and multivariable flight guidance for airbreathing launch vehicles," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 6, 1997, pp. 1157–64.

[7]Schmidt, D. K. and Velapoldi, J. R., "Flight Dynamics and Feedback Guidence Issues for Hypersonic Airbreathing Vehicles," AIAA Paper 99-4122, Aug. 1999.

[8]Davidson, J., Lallman, F., McMinn, J. D., Martin, J., Pahle, J., Stephenson, M., Selmon, J., and Bose, D., "Flight Control Laws for NASA's Hyper-X Research Vehicle," *AIAA Guidance, Navigation, and Contronol Conference and Exhibit*, 1999.

[9]Tournes, C., Landrum, D. B., Shtessel, Y., and Hawk, C. W., "Ramjet-powered reusable launch vehicle control by sliding modes," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 3, 1998, pp. 409–15.

[10]Doman, D. B. and Ngo, A. D., "Dynamic inversion-based adaptive/reconlgurable control of the X-33 on ascent," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 2, 2002, pp. 275–84.

[11]Wang, Q. and Stengel, R., "Robust nonlinear control of a hypersonic aircraft," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 577–85.

[12]Marrison, C. and Stengel, R., "Design of robust control systems for a hypersonic aircraft," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, 1998, pp. 58–63.

[13]Xu, H., Mirmirani, M., and Ioannou, P., "Adaptive sliding mode control design for a hypersonic flight vehicle," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 5, 2004, pp. 829–38.

[14]Sigthorsson, D. O., Serrani, A., Yurkovich, S., Bolender, M. A., and Doman, D. B., "Tracking Control for an Overactuated Hypersonic Air-Breathing Vehicle with Steady State Constraints," AIAA Paper 2006-6558, Aug. 2006.

[15]Groves, K. P., Serrani, A., Yurkovich, S., Bolender, M. A., and Doman, D. B., "Anti-Windup Control for an Air-Breathing Hypersonic Vehicle Model," AIAA Paper 2006-6557, Aug. 2006.

[16]Parker, J. T., Serrani, A., Yurkovich, S., Bolender, M. A., and Doman, D. B., "Approximate Feedback Linearization of an Air-breathing Hypersonic Vehicle," AIAA Paper 2006-6556, Aug. 2006.

[17]Kristic, M., Kanellakopoulous, I., and Kokotovic, P. V., *Nonlinear Adaptive Control Design*, New York: Wiley, 1995.

[18]Jankovic, M., Sepulchre, R., and Kokotovic, P., "Constructive Lyapunov stabilization of nonlinear cascade systems," *IEEE Transactions on Automatic Control*, Vol. 41, No. 12, 1996, pp. 1723–35.

[19]Isidori, A., *Nonlinear Control Systems II*, Springer-Verlag, Berlin, 1999.

[20]Schierman, J., Ward, D., Hull, J., Gandhi, N., Oppenheimer, M., and Doman, D., "Integrated adaptive guidance and control for re-entry vehicles with flight-test results," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 6, 2004, pp. 975–88.

[21]Khalil, H. K., "Adaptive output feedback control of nonlinear systems represented by input-output models," *IEEE Transactions on Automatic Control*, Vol. 41, No. 2, 1996, pp. 177–88.

[22]Teel, A. R., "A nonlinear small gain theorem for the analysis of control systems with saturation," *IEEE Transactions on Automatic Control*, Vol. 41, No. 9, 1996, pp. 1256–70.

[23]Teel, A. R. and Praly, L., "Tools for semiglobal stabilization by partial state and output feedback," *SIAM Journal on Control and Optimization*, Vol. 33, No. 5, 1995, pp. 1443–88.

[24]Williams, T., Bolender, M. A., Doman, D. B., and Morataya, O., "An Aerothermal Flexible Mode Analysis of a Hypersonic Vehicle," AIAA Paper 2006-6647, Aug. 2006.

[25]Bolender, M. A. and Doman, D. B., "A Non-Linear Model for the Longitudinal Dynamics of a Hypersonic Air-breathing Vehicle," AIAA Paper 2005-6255, Aug. 2005.

[26]Chavez, F. and Schmidt, D., "Analytical Aeropropulsive/Aeroelastic Hypersonic-Vehicle Model with Dynamic Analysis," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 6, 1994, pp. 1308–19.

[27]Bolender, M. A. and Doman, D. B., "Flight Path Angle Dynamics of Air-breathing Hypersonic Vehicles," AIAA Paper 2006-6692, Aug. 2006.

[28]Khalil, H., *Nonlinear Systems*, Prentice Hall, Upper Saddle River, NJ, 3rd ed., 2002.